2010 8th IEEE International Conference on
Control and Automation
Xiamen, China, June 9-11, 2010

Input Time Delay Systems Identification Via Wavelet approach

M. Najafi, M. Zekri, and J. Askari

Abstract— **In this paper, a new and effective method is presented for on-line parameters identification of systems with unknown time delay. Our method is based on wavelet approach for time delay identification and Gradient or RLS algorithm is used for estimating transfer function parameters. So two parametric models of system are presented that one of them is used for time delay estimation and other one is used for parameters identification. The effectiveness of this method has been demonstrated through simulation.**

I. INTRODUCTION

Now days system identification is an active area of automatic control. A considerable number of identification methods have been reported in [1] and [2], and they are generally classified into parametric and non-parametric methods. Transfer function parameters estimation is one of the most popular parametric methods. However, most of the existing methods for transfer function identification do not consider the process delay or just assume knowledge of the delay [3].

Time delay is a common phenomenon in many industrial processes due to material and energy transportation lags, measurement delay, and so on [4]. In many industrial systems the time delay and other parameters of the system are unknown. Thus, there has been continuing interest in identification of delay processes with unknown delays. One method is based on determining delay value from the process reaction curve obtained by injecting a step signal to the system [5]. This approach is off-line and is not suitable for systems with time-varying parameters.

Three main approaches have been presented for on-line identification of systems with unknown time delay. The first one is based on approximating the time delay with a rational transfer function such as polynomial approximation [6], Padé approximation [7] and Laguerre expansion [8]. The second is rooted in identification of a lower order model of the unknown system. It is inspired by Astrom's method [9] where the closed-loop system is brought into limit cycle oscillations by switching the closed-loop to a relay excitation and extrapolating the critical gain and frequency. The third approach is based on recursive estimation of system parameters [10].

[14] and [15] suggest two parametric model that one of them is used for parameters identification and other one for time delay estimation. [15] utilizes a comparison method for time delay estimation and Gradient algorithm for parameter identification. But in [14] the correlation method is used for time delay identification and recursive least square (RLS) is applied for parameter identification. If the correlation is described by the following equation

$$R(\tau) = \sum y_f(t) u_f(t-\tau) \tag{1}$$

where

$$y_f = u_f . e^{-sd} \tag{2}$$

It is a fact that $R(\tau)$ is maximum value when $\tau = d$. [14] suggests two models of system that are the same with d second difference. Then it calculates (1) for all probable values of d , ($\forall \tau \in [0, d_{\max}]$) and chooses $d = \tau^*$ when $R(\tau^*)$ is maximum.

Correlation method is really one of the best approach for time identification but in this method wee need to calculate the correlation for each τ and find the maximum of $R(\tau)$ by searching. It means that this method needs too many calculations for each step of estimation.

For improving this method, we can use wavelet transform of each model. Wavelet is correlation result with a signal and reference signal. Therefore by using wavelet transform, we can estimate delay by comparing between wavelet transform of two signals or two models. This point is main idea in this paper.

In this note, a new and effective method is presented for on-line identification of systems with unknown time delay. This method is based on two parametric models of system that one of them is defined for transfer function parameters estimation and another one is defined for delay value estimation. Gradient algorithm or (RLS) is used for estimating transfer function parameters and a wavelet method is proposed for estimating delay value.

This paper is organized as follows: In section 2, our method is described for identification of time delay between two same signals by wavelet approach. Practically points for delay estimation are presented in section 3. A method

Majdeddin Najafi is PH.D student in the Electrical and Computer Engineering Department, Isfahan University of Technology, Isfahan, Iran (phone:98-913-3089117, e-mail: najafi_majd@ec.iut.ac.ir).

Maryam Zekri is Assistant Professor in the Electrical and Computer Engineering Department, Isfahan University of Technology, Isfahan, Iran (e-mail: mzekri@cc.iut.ac.ir).

Javad Askari is Assistant Professor in the Electrical and Computer Engineering Department, Isfahan University of Technology, Isfahan, Iran (e-mail: j-askari@cc.iut.ac.ir).

978-1-4244-5196-8/10/$26.00 ©2010 IEEE

for identification of input-delay systems with unknown time delay is introduced in section 4. In sections 5, a simulation example is provided to verify the performance of the proposed method. Finally a brief conclusion is drawn in section 6.

II. IDENTIFICATION OF TIME DELAY WITH TWO SIGNALS

In this part, we assume that we have two signals. One of them is the reference signal and other one is similar reference signal with time delay. Then we try to estimate time delay with a wavelet approach. Consider two signals $\hat{z}$ and z that

$\hat{z}(t) = z(t-d)$

That z is the reference signal and $\hat{z}$ is the delayed signal. It is clear that the wavelet transform of those signals is the same with d second shift. The wavelet transform is described by

$$W_z(a,b) = \int_{-\infty}^{\infty} z(t)\Psi_{a,b}(t)dt \tag{3}$$

where $\Psi_{a,b}(t)$ is dilated and translated version of a mother wavelet function:

$$\Psi_{a,b}(t) = \frac{1}{\sqrt{a}}\Psi(\frac{t-b}{a}) \tag{4}$$

Also the function $\hat{z}(t)$ can be constructed by the inverse wavelet transform

$$z(t) = \frac{1}{C_\Psi}\iint W_z(a,b)\Psi_{a,b}\frac{da.db}{a^2} \tag{5}$$

where C_Ψ is admissible condition:

$$C_\Psi = \int_{-\infty}^{\infty}\frac{\left|\hat{\Psi}(w)\right|^2}{w}dw \tag{6}$$

For example, consider that the wavelet transforms of z and $\hat{z}$ are like the figures (1) and (2).

The wavelet transforms are calculated where $b = k \times 0.1$ (k is integer) and $a = 1,2,\ldots,16$. It is clear that figures (1) , (2) are the same but with 10 points (1 second) shift. For example the 65th shift of figure (1) is conformity with the 75th shift of figure (2).

Therefore we must to estimate this shift for estimating the time delay. The following we propose an equation in order to estimate the time delay ($\hat{d}(t)$):

$$\hat{d}(t) = \{b - b^* \Big/ \arg\min_b \int_0^{a_{\max}} \left|W_z(a,b^*) - W_{\hat{z}}(a,b)\right| \quad \forall\ \left[b^*, b^* + d_{\max}\right] \tag{7}$$

Where b^* is the arbitrary shift of wavelet transform of without delayed signal, and $d_{\max}$ is the maximum expended delay ($d = [0, d_{\max}]$).

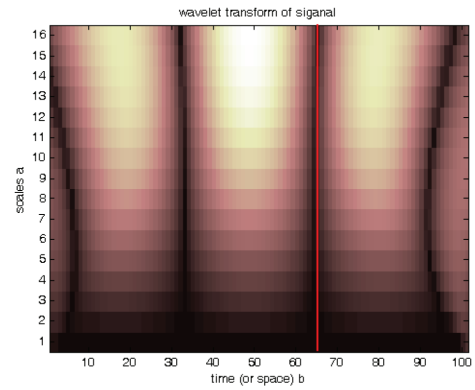

Figure (1)

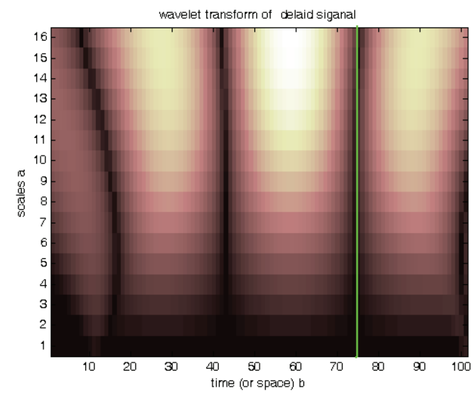

Figure (2)

Proof:

$$\int_0^{a_{\max}} \left|W_z(a,b^*) - W_{\hat{z}}(a,b)\right| da = \int_0^{a_{\max}} \left|\int_{-\infty}^{\infty} z(t)\psi(a,b^*)dt - \int_{-\infty}^{\infty} z(t-d)\psi(a,b)dt\right| da \tag{8}$$

$$= \int_0^{a_{\max}} \left|\int_{-\infty}^{\infty} z(t)\frac{1}{\sqrt{a}}\psi(\frac{t-b^*}{a})dt - \int_{-\infty}^{\infty} z(t-d)\frac{1}{\sqrt{a}}\psi(\frac{t-b}{a})dt\right| da$$

$$= \int_0^{a_{\max}} \left|\int_{-\infty}^{\infty} f(x)\frac{1}{\sqrt{a}}\psi(\frac{x-b^*}{a})dx - \int_{-\infty}^{\infty} f(x-d)\frac{1}{\sqrt{a}}\psi(\frac{x-b}{a})dx\right| da`$$

It follows from the changing variable

$$\int_{-\infty}^{\infty} z(t-d)\frac{1}{\sqrt{a}}\psi(\frac{t-b}{a})dt \overset{x=t-d}{=} \int_{-\infty}^{\infty} z(x)\frac{1}{\sqrt{a}}\psi(\frac{x+d-b}{a})dx$$

We have

$$= \int_0^{a_{\max}} \left|\int_{-\infty}^{\infty} z(x)\frac{1}{\sqrt{a}}\psi(\frac{x-b^*}{a})dx - \int_{-\infty}^{\infty} z(x)\frac{1}{\sqrt{a}}\psi(\frac{x-(b-d)}{a})dx\right| da$$

It is clear that (8) is zero at $b = b^* + d$.

III. ONLINE DELAY ESTIMATION

In pervious section, we proposed a method for time delay estimation between two hold similar signals. But in this part we want to online estimate the delay. So, there are some problems about online estimation that are described follow:

A. Reduce the computation volume

The first problem is the high computation volume when the time increases. It means, we have to calculate wavelet transform at all scales and shift. For each wavelet transform we have

$$W_z(a,b) = \int_{-\infty}^{\infty} z(t)\Psi_{a,b}(t)dt \qquad a \in [1, a_{\max}] \qquad (9)$$
$$b \in [0,t], b = k.\Delta t$$

If (9) is calculated in each Δt step time, we have to compute $a_{\max} \times (t/\Delta t)$ integral in each step time for a wavelet transform when the time increases. According to (7), we realize that it is enough to calculate wavelet transform only at $b = b^*$ for reference signal and $b \in [t - d_{\max}, t]$ for delayed signal. In this way, the computation volume is constant in the all of time. Therefore we have to calculate $a_{\max}$ integral for reference signal and $a_{\max} \times (d_{\max}/\Delta t)$ integral for reference signal.

B. Choose a windows of signals

To reduce the computation volume of wavelet transform, we can decrease the limit integral in (7). So we can choose a window of each signal instead of all the interval time of signal. For online identification, it is necessary to choose this window near the end of the signal time.

For delay estimation we have to compare two signals for $d_{\max}$, second. Therefore the width of this window at least must be equal to $d_{\max}$. But if we want to compare wavelet transform of those signals, should those wavelets is good approximate of real wavelets of signals. So this the width of window should be chosen a multiple of $d_{\max}$.

Also for online identification, we have to select this window near the real time. Finally, we propose to choose the width of window like follow

$$\Delta w_d \approx 50.D_{\max} \qquad t - 50.D_{\max} \le w_d \le t \qquad (10)$$

C. Chose optimal point for b^*

To develop better estimation with less error, it is necessary we choose the best point for b^* in the reference signal. If we want to online estimation, we should select the value of b^* near the real time of reference signal windows. But we have to obviate two limitations.

At first, it is the fact that delayed signal is lag from reference signal for $d_{\max}$ in max state. So b^* should have $d_{\max}$ second distance of real time at least.

In other limitation the first and end of wavelet transform may be taken effect of the cut of signal and damaged. This problem is shown in figure (3).

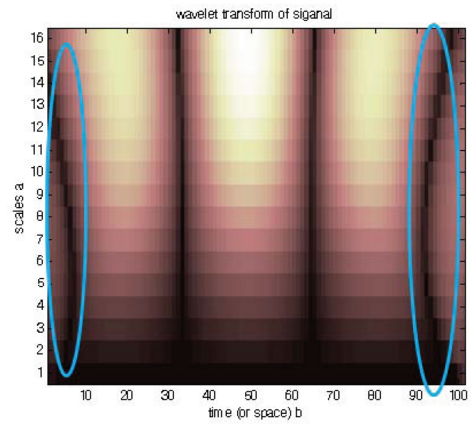

Figure (3)

Therefore, we suggest choose $b^* = t - D_{\max} - \varepsilon$, where t is real time and ε is the bound of damaging wavelet that is created by cut signal.

IV. IDENTIFICATION OF INPUT-DELAY SYSTEMS WITH UNKNOWN TIME DELAY

Consider a linear input-dealy system described by

$$y(s) = \frac{A(s).e^{-ds}}{B(s)}.u = \frac{A(s)}{B(s)}.u(t-d)$$

Where $u(t)$ and $y(t)$ are input and output signals of the plant, respectively. The time delay $d \in \Re^+$ is assumed to be unknown, but it is assumed that the upper bound of delay value is known ($0 \le d \le d_{\max}$). Our objective is to estimate the unknown parameters $\theta = [b_m, b_{m-1}, \ldots, b_0, a_{n-1}, \ldots, a_0]^T$ of the $A(s), B(s)$ and unknown delay value d based on input and output data measurements.

In order to estimate parameter vector θ and delay value, d , two estimates $\hat{z}_1$ of z_1 and $\hat{z}_2$, $\hat{z}_3$ at each time t are generated, respectively. As in previous section, the estimate $\hat{z}_1$ are defined with parametric model

$$z_1 = \theta^T(t) \times \phi_1 \qquad (11)$$
$$\hat{z}_1 = \hat{\theta}^T(t) \times \phi_1 \qquad (12)$$

where

$$\phi_1 = \frac{1}{\Lambda(s)}[s^m e^{-\hat{d}s}u, s^{m-1}e^{-\hat{d}s}u, \ldots, e^{-\hat{d}s}u, \qquad (13)$$
$$-s^{n-1}y, -s^{n-2}y, \ldots, -y]^T$$

In (13), $\Lambda(s) = s^n + \lambda_1 s^{n-1}, \ldots, \lambda_n$ is a monic Hurwitz polynomial, s is Laplace and $\hat{d}$ is estimation delay of system that is described following.

The identification error is denoted as $e_1 = z - \hat{z}_1$ and $\hat{\theta}(t)$ is generated by gradient adaptive law or RLS method. According to gradient adaptive low we have

$$\dot{\hat{\theta}}(t) = \Gamma e_1 \phi_1 \qquad (14)$$

where Γ is a constant positive definite matrix.

We can also used recursive least square (RLS) method to estimate $\hat{\theta}(t)$ estimation. In this way we consider the following group equations:

$$\hat{\theta}(t_{i+1}) = \hat{\theta}(t_i) + k(t_i)e(t_i)$$

$$k(t_i) = \frac{P(t_{i-1})\phi_1(t_i)}{\lambda(t_i) + \phi_1^T(t_i)P(t_i)\phi_1(t_i)} \quad (15)$$

$$P(t_i) = \frac{[1 - k(t_{i-1})\phi_1^T(t_i)]P(t_{i-1})\phi_1(t_i)}{\lambda(t_i)}$$

Where $k(t_i)$, $P(t_i)$ and $\lambda(t_i)$ are the estimator gain, covariance matrix and the forget factor, respectively.

Gradient algorithm are continues, but in RLS method, we should calculate $\hat{\theta}(t_{i+1})$, $k(t_i)$ and $P(t_i)$ step by step. In simulation example we use both two methods and compare with each other.

The estimation of the time delay ($\hat{d}(t)$) is obtained by $\hat{z}_2$ and $\hat{z}_3$ that are defined as

$$\hat{z}_2 = \frac{\hat{B}(s)}{\Lambda_1(s)}u \quad (16)$$

$$\hat{z}_3 = \frac{\hat{A}(s)}{\Lambda_1(s)}y \quad (17)$$

where $\hat{A}(s), \hat{B}(s)$ are estimation of $A(s), B(s)$ that is calculated by (14) or (15). If $A(s) = \hat{A}(s), B(s) = \hat{B}(s)$ and $\Lambda_1(s) = s^n + \lambda_1 s^{n-1},..., \lambda_n$ is a monic Hurwitz polynomial. It is clear that

$$\hat{z}_3 = \hat{z}_2.e^{-sd} \quad (18)$$

Two estimation $\hat{z}_2, \hat{z}_3$ are same with d seconded delay. Therefore we can use the wavelet technique for delay estimated by (7).

Finally, in each step of estimation, we calculate (13) by using input, output and estimation delay that estimated in pervious step. So is estimated parameter of system by (14) or (15). According to estimated parameter, is calculated (16) and (17) and identify delay by (7).

V. Simulation Results

To illustrate the effectiveness of the proposed method, let us consider the following system described b

$$y(s) = \frac{1}{s+2}e^{-0.6s}u(s)$$

For identification this system consider

$$z_1 = \frac{s}{\Lambda(s)}y(s) = \theta.\Phi(s)$$

where

$$\theta = [1 \quad 2] \qquad \Phi = \frac{1}{s+1}\left[u.e^{\hat{d}.s} \quad -y\right]$$

$$\Lambda(s) = s + 1$$

and $\hat{d}$ is estimated time delay. Therefore the estimation model is

$$z_1 = \frac{as}{s+b}y(s) = \hat{\theta}.\Phi(s)$$

where

$$\hat{\theta} = [a \quad b]$$

Also two models are defined for delay estimation:

$$\hat{z}_2 = \frac{s+b}{\Lambda_1(s)}u(s) \qquad \hat{z}_3 = \frac{as}{\Lambda_1(s)}y(s)$$

where a and b are estimated parameters and $\Lambda(s) = s + 5$

In this example we consider the maximum of delay $d_{\max} = 1$ and the width of window of each signal about 50 second ($t - 50 \le w_d \le t$).

Figures (4), (5) show the parameter and delay estimation by using the Gradient algorithm and wavelet approach

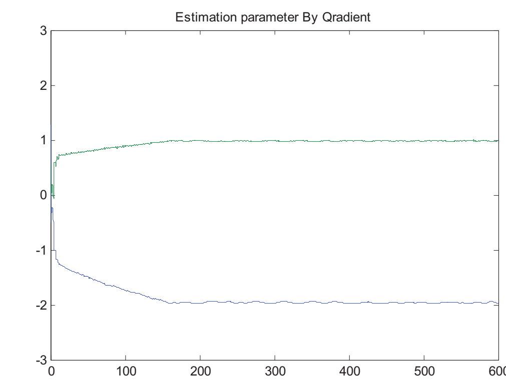

Figure (4)

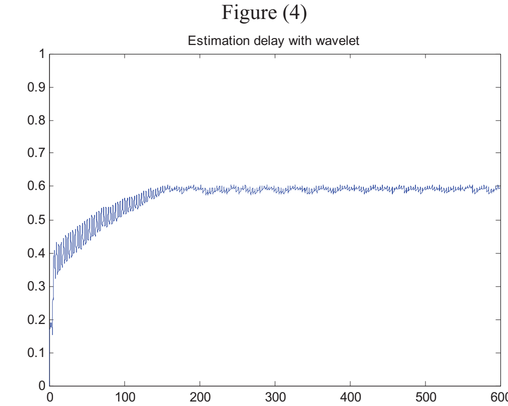

Figure (5)

Figures (6), (7) show the parameter and delay estimation by using the RLS algorithm and wavelet approach.

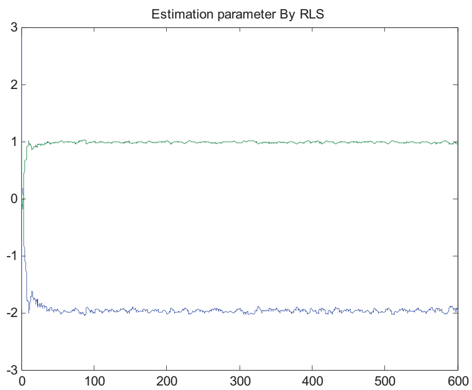

Figure (6)

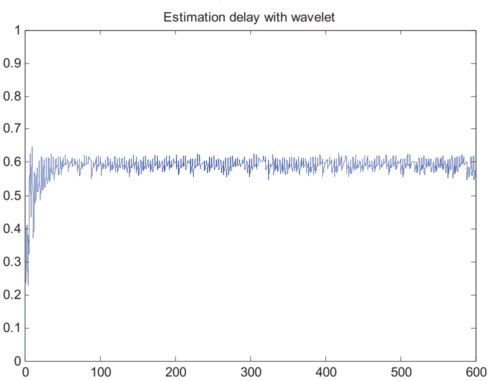

Figure (7)

According to figures (4),(5),(6) and (7), both of methods are estimate system, error estimation converge to zero and parameter of the system and estimated time delay converge to real value. Since the speed of RLS algorithm estimation usually is faster than Gradient, so time delay estimation by RLS is also faster.

VI. CONCLUSION:

This paper has presented a new and effective method for on-line identification of systems with unknown time delay. In this method we defined two separately parametric models of system for transfer function parameters estimation and delay value estimation. Gradient or RLS algorithm can be used for estimating transfer function parameters and a wavelet algorithm is suggested for estimating delay value. A simulation example is given to show the capability and the performance of the proposed method. We believe this method would be very useful for computer based identification and control of industrial unknown systems with unknown time delay.

Reference:

[1] L. Ljung, *System Identification: Theory for the user*. Upper Saddle River, NJ: Prentice-Hall, 1999.

[2] O. Nells, *Nonlinear System identification,* Springer, 2001

[3] Q.- G. Wang, X. Guo, Y. Zhang, "Direct identification of continuous time delay systems from step responses", *J. Process Control*, Vol.11, No., pp.531-542, 2001.

[4] M. Malek-zavarei, M. Jamshidi, *Time delay systems: Analysis, Optimization and Applications,* North-Holland, Amsterdam, 1987.

[5] K. R. Sundaresan, C. C. Prasad, and P. R. Krishnaswamy, "Evaluating parameters from process transients," *I & EC Process Design Development*, vol. 17, no. 3, pp. 237–241, 1978.

[6] P. J. Gawthrop, M. T. Nihtila, "Identification of time-delays using polynomial identification method", *System and Control Letters,* Vol.5, pp.267-271, 1985.

[7] C. E. D. Souza, G.C. Goodwin, D. Q. Mayne, M. Palaniswami, "An adaptive control algorithm for linear systems having unknown time delay", *Automatica*, Vol.24, No.3, pp.327-341, 1988.

[8] R. Malti, S.B. Ekongolo, "Dynamic SISO and MIMO system approximation based on optimal Laguerre models", *IEEE Trans. Aut. Control*, Vol.43, No.9, pp.1318-1323, 1998.

[9] K. J. Astrom and T. Hagglund, "Automatic tuning of simple regulators with specifications on phase and amplitude margins," *Automatica*, vol. 20, pp. 645–652, 1984.

[10] P. J. Gawthrop, M. T. Nihtila, and A. B. Rad, "Recursive parameter estimation of continuous systems with unknown time delay," *Control Theory Adv. Technol.*, vol. 5, no. 3, pp. 227–248, 1989.

[11] P. A. Ioannou, J. Sun, *Robust Adaptive Control*, Prentice-Hall, New Jersey, 1996.

[12] S. Sastry, M. Bodson, *Adaptive control Stability, Convergence, and Robustness*, Englewood Cliffs, New Jersey, 1989.

[13] K. J. Astrom, B. Wittenmark, *Adaptive control,* Addison-Wesley, New York, 1995.

[14] A. B. Rad, W. L. Lo, and K. M. Tsang "Simultaneous online identification of rational dynamicsand time delay: A correlation-based approach",*IEEE Trans on Control Systems Technology*, VOL. 11, NO. 6, NOV 2003 957

[15] M. Najafi, M. Kamali, J. Askari, "Parametric identification of input-delay systems with unknown time delay", *Proc of the 26th Chinese Control Conf*, July 26-31, 2007